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The entropy of the microwave background and the acceleration of the universe

Willy Fischler, Amit Loewy and Sonia Paban

Department of Physics
University of Texas, Austin, TX 78712, USA
E-mail: fischler@physics.utexas.edu, loewy@physics.utexas.edu, paban@utaphy.ph.utexas.edu

Abstract: If the present acceleration of the universe is due to a cosmological constant, λ, then the entropy of the microwave background is bounded. It cannot exceed $\lambda^{-3/4} \sim 10^{91}$, which is much less than the entropy of empty de Sitter space $\lambda^{-1} \sim 10^{122}$. This is due to the limited efficiency of storing entropy by local field theoretical degrees of freedom. The observed entropy of the microwave background is of $O(10^{85})$.

Keywords: Classical Theories of Gravity, Black Holes.

Contents

1. Introduction

Recent observations of supernovae indicate that the universe is accelerating [1, 2, 3]. This suggests that, at present, the energy density of the universe is dominated by a fluid with an equation of state $p < -\frac{1}{3}\rho$. We will assume in this paper that the source of the observed acceleration is a positive cosmological constant

$$p = -\rho = -\lambda\,. \tag{1.1}$$

We will then argue that in this case, the entropy stored in local field theoretical degrees of freedom cannot exceed a bound set by a power of the cosmological constant. The upper bound on this entropy scales like $\lambda^{-3/4}$, which is parametrically smaller than the entropy of empty de Sitter space, $S_{DS} \sim \lambda^{-1}$ [4]. We will refer in the rest of the paper to the entropy of the microwave background, although the discussion applies to the entropy of any field theoretical degree of freedom.

The physics that underlies the bound, in a nutshell, is the limited efficiency with which a fluid described by local field theoretical degrees of freedom stores entropy in space. This capacity to store information, as seen by local observers, depends on the equation of state. It was shown [6] that the entropy of a fluid with an equation of state $p = \kappa\rho$ scales as a power of the linear size R,

$$S \sim R^{\,3-\frac{2}{1+\kappa}}\,. \tag{1.2}$$

This result was obtained in the absence of a cosmological constant. We will show that any attempt, when there is no cosmological constant, to squeeze more entropy in a region of fixed size R leads to the formation of black holes. This implies that for radiation, $\kappa = 1/3$, a given region of space with linear size R can only accommodate an entropy of $O(R^{3/2})$. This is rather inefficient when compared to the storage capabilities of a black hole, or a fluid with $\kappa = 1$. Indeed, in the latter cases the entropy scales like the area, R^2.

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One might then wonder about what happens to the "storage of entropy" in the presence of a positive cosmological constant. We will argue that in order for such a space-time to asymptote to de Sitter space, there is a bound on the amount of stored entropy, that a local observer can ever measure. This local observer eventually perceives this space-time as a hot cavity [7] of finite size $\lambda^{-1/2}$ in which the entropy of the microwave background never exceeds $\lambda^{-3/4}$. The bulk of the paper is devoted to argue that any attempt to store more entropy than the bound allows, meets with black hole formation or a change of the equation of state to a stiffer one, or more probably imprints itself on the event horizon of de Sitter space, and in some cases the universe ends up in a big crunch.

The paper is organized as follows: In section 2, as a warm-up, we discuss what happens in the contracting phase of a space-time with a cosmological constant and a perfect fluid, as a function of the entropy of the fluid. In section 3, we discuss how efficient a fluid in equilibrium can be in packing entropy. We then use in section 4 the Tolman-Oppenheimer-Snyder (TOS) model [8] in a way that differs from its original purpose. In this paper, we use the TOS model in the presence of a cosmological constant [9] to give us a handle on the amount of entropy inside a "star". This will allow us to draw further evidence for our bound. We end with conclusions and an appendix where a preliminary setup in the "O-gauge" [16] can be found.

2. Contracting phase of a space-time with a fluid and a cosmological constant

In this section we will consider the case of a homogeneous universe with a positive cosmological constant and a perfect fluid. We will show that any attempt to store more information, during the contracting phase of this cosmology, than what is allowed by a bound on the entropy, leads to a big crunch.[1] Consider the Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = dt^2 - a(t)^2\left(d\theta^2 + \sin^2\theta d\Omega_2^2\right) . \tag{2.1}$$

The equation that the scale factor obeys is,

$$\left(\frac{\dot a}{a}\right)^2 + \frac{1}{a^2} = \frac{\lambda}{3} + \rho(a)\,, \tag{2.2}$$

where $\rho(a)$ is the energy density of the fluid.[2] We will use a fluid with a linear equation of state, $p = \kappa\rho$. For a given κ the energy density obeys

$$\rho(a) = \frac{\rho(a_0)\, {a_0}^{3(1+\kappa)}}{a^{3(1+\kappa)}}\,. \tag{2.3}$$

By using the thermodynamic relation between the energy and entropy densities,

$$\rho \sim \sigma^{1+\kappa}\,, \tag{2.4}$$

[1] For a discussion on the endpoint of a big crunch, see [10].

[2] For the remainder of the paper we will work in Planck units.

it is then convenient for our purposes to rewrite the energy density as,

$$\rho(a) = \frac{{S_0}^{1+\kappa}}{a^{3(1+\kappa)}}\,, \tag{2.5}$$

where S_0 is the entropy associated with the fluid when the scale factor is a_0. Using this last equation, we can then rewrite (2.2) as

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{a^2} = \frac{\lambda}{3} + \frac{{S_0}^{1+\kappa}}{a^{3(1+\kappa)}}\,. \tag{2.6}$$

This equation implies that in order to avoid a big crunch, $a \to 0$, S_0 should be bounded from above by

$$S_0 < C_\kappa \lambda^{-\frac{1+3\kappa}{2(1+\kappa)}}\,, \tag{2.7}$$

where C_κ is a positive number that depends on κ and microscopic physics of the fluid.[3] For radiation the bound reads

$$S_0 < C_{1/3}\lambda^{-3/4}\,, \tag{2.8}$$

where $C_{1/3}$ is a coefficient of $O(1)$.

The bound for the entropy given by (2.7), has an intriguing interpretation. The power law is exactly the one that is obtained if one answers the question about the maximum amount of entropy in the fluid, at the threshold of black hole formation, that can fit in a box of linear size, $\lambda^{-1/2}$, i.e. the size of the de Sitter horizon.

3. Efficiency of entropy packing

In this section we will give a heuristic derivation of the scaling law of the entropy of a fluid as a function of the size of the region to which it is confined. We will assume that a perfect fluid contained in a box of size R has a linear equation of state $p = \kappa\rho$. We will estimate the entropy versus box size as one tries to pack entropy to the verge of black hole formation. We will assume that a single mode but no more than a single mode of the fluid, with energy ϵ occupies a volume ϵ^3 in Planck units. The number density of modes is

$$n(\epsilon) = \epsilon^{-3}\,. \tag{3.1}$$

The energy density as a function of ϵ is,

$$\rho(\epsilon) = \epsilon^{-2}\,. \tag{3.2}$$

Using the thermodynamics relation between the entropy density σ and the energy density ρ,

$$\sigma(\epsilon) \sim \rho(\epsilon)^{\frac{1}{1+\kappa}}\,, \tag{3.3}$$

we obtain a relation, at the threshold of black hole formation [12], between the entropy S

[3] A similar bound on the entropy as a function of the cosmological constant for closed universes was found years ago by Gibbons [11] in a completely different context.

and the size R of the region where the fluid is contained.

$$S(R) \sim \int_0^R d\epsilon\, \epsilon^2 \sigma(\epsilon) \sim R^{\,3-\frac{2}{1+\kappa}}\,. \tag{3.4}$$

Notice that this is the exact same scaling law for the entropy that was derived from the Tolman-Oppenheimer-Volkov (TOV) equations [6] describing a spherically symmetric perfect fluid in equilibrium.[4]

It seems reasonable to conclude, based on the earlier heuristic derivation of the scaling law, that any attempt to squeeze more entropy in a given volume beyond the bound (2.7) leads to black hole formation. This picture seems consistent with our present understanding that a black hole is the most efficient environment at packing entropy.

The previous discussion on efficient entropy packing, is modified by the presence of a cosmological constant. Additional distinction should be made between the contracting and expanding parts of the FRW geometry. In the contracting phase black hole formation is enhanced, and the bound on the entropy remains the same. In the expanding case, when the size of the region containing the fluid becomes of $O(\lambda^{-1/2})$ the fluid is subjected to a pull due to the cosmological constant, which makes it increasingly harder to increase the entropy in that region beyond the bound discussed in the absence of λ. However any attempt to violate the bound is accompanied by a repackaging of the excess entropy until one reaches the "Bekenstein bound".

In order to provide additional intuition about the bound for the entropy of a fluid in the presence of a cosmological constant, we will devote our next discussion to studying a modified version of the TOS model [8]. The TOS model was originally designed to describe the collapse of a spherical star made of dust. The interior of the star is modeled by a FRW geometry, whereas the exterior (using Birkhoff's theorem) is a static Schwarzchild geometry.

4. A generalized TOS model

The TOS model will be modified in what follows, by adding a cosmological constant [9]. Our intention is to view the star from the outside as the amount of entropy on the inside is tuned to eventually violate the aforementioned bound. In the setup of this generalized TOS model, we will closely follow some of the analysis by Markovic and Shapiro [9]. This study will give us evidence for our claim that the entropy in a universe with a cosmological constant is bounded by an amount that is parametrically smaller than the Bekenstein bound.

Basically, the TOS model describes the dynamics of a ball of dust in the presence of a positive cosmological constant. The region inside the ball is described by a (truncated) FRW geometry,

$$ds^2 = d\tau^2 - a(\tau)^2\left(\frac{dx^2}{1-kx^2} + x^2 d{\Omega_2}^2\right), \tag{4.1}$$

where k determines the curvature of the spatial part of the geometry. The x coordinate take values in the region $0 \le x \le X$.

[4] The derivation of the entropy at the threshold of black hole formation can be easily generalized to an arbitrary number of dimensions and $\frac{1}{1-D} < \kappa \le 1$. The result reproduces the scaling obtained using the TOV equations in D dimensions

The scale factor $a(\tau)$ satisfies

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\lambda}{3} + \frac{\xi}{F(X)}\frac{S(a_0)}{a^3}, \tag{4.2}$$

where ξ is a fixed parameter that has dimensions of mass, and depends on microscopic physics. This parameter appears in the relation between the energy density and the entropy density of the dust

$$\rho = \xi\sigma, \tag{4.3}$$

and

$$F(X) = \int_0^X \frac{dx\ x^2}{\sqrt{1-kx^2}}. \tag{4.4}$$

The generalized Birkhoff theorem [13] guarantees that the space-time outside the sphere is Schwarzchild-de Sitter (SdS)

$$ds^2 = \left(1 - \frac{2M}{r} - \frac{\lambda}{3}r^2\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{\lambda}{3}r^2\right)} - r^2 d{\Omega_2}^2. \tag{4.5}$$

In order to complete the description of the space-time geometry, we need to match the 3-geometry across the interface between the dust and the exterior space-time, which is at $r = R(\tau)$ using the the exterior coordinate patch, and at $x = X$ using the interior coordinate patch. This is achieved by imposing the continuity of the surface's 3-metric

$$ds_3^2 = d\tau^2 - R^2 d{\Omega_2}^2, \tag{4.6}$$

where $R(\tau) = Xa(\tau)$, and then imposing Israel's junction conditions [14]. As shown in [9] these conditions lead to an equation for $R(\tau)$

$$\dot{R}^2 + f(R) = 1 - kX^2, \tag{4.7}$$

where

$$f(R) = 1 - \frac{2M}{R} - \frac{\lambda R^2}{3}. \tag{4.8}$$

We want to remind the reader that we are exploring the consequences of injecting more entropy than the bound given in (2.7). With that in mind we will take an S_0 in (4.2) that exceeds the bound by requiring $S_0 > \lambda^{-1/2}$ parametrically.

By comparing equations (4.7), (4.8) and (4.2) one finds a relation between the "mass" as seen from the outside, and the amount of entropy, inside the "star"

$$M = \xi\frac{X^3}{F(X)}S(a_0), \tag{4.9}$$

where the ratio $X^3/F(X)$ is bounded below by a number of $O(1)$, independent of λ.

If we now impose that the entropy exceeds our bound, $S_0 > \lambda^{-1/2}$ parametrically, we find that the "mass" is parametrically larger than $\lambda^{-1/2}$, which in turn implies that the

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function $f(r)$ that appears in the SdS metric changes sign. At this point, the view of the universe as a cavity no longer applies.

$$f(r) = 1 - \frac{2M}{r} - \frac{\lambda r^2}{3} < 0\,. \tag{4.10}$$

This implies an exchange of roles between the radial and time coordinates in the description of the space-time geometry. The metric can then be rewritten by introducing $T \equiv r$ or $T \equiv -r$ and $y \equiv t$

$$ds^2 = \frac{1}{g(T)} dT^2 - g(T) dy^2 - T^2 d\Omega_2^2\,, \tag{4.11}$$

where

$$g(T) = \frac{\lambda T^2}{3} + \frac{2M}{T} - 1\,. \tag{4.12}$$

The space-time geometry is now time dependent. The inside of the "star" and the outside geometry either both collapse to a big crunch or expand from a big bang singularity. The outside geometry looks like a contracting or expanding hyper-cylinder. This is rather different from the case where the entropy of the "star" satisfies the bound.

We do not expect that an increase of the entropy beyond the bound, entails such drastic changes. Instead, we believe that the excess entropy will be stored, some in small black holes, but most we believe stored on the de Sitter horizon. The case concerning the behavior of a "star" has to be contrasted with the case of black holes where the entropy storage is maximal. A somewhat related phenomenon is the case of the repulsion between two black holes in de Sitter space [15], with respective areas smaller than a bound allowed by de Sitter space, but whose merger would have created a black hole with size larger than the bound

The main point that we are trying to argue in this paper is that the entropy in radiation that a local observer can measure cannot exceed a bound, that is smaller parametrically than the Bekenstein bound. Any excess entropy gets repackaged. So far we have provided circumstantial evidence for the bound and what remains to be shown is how this bound emerges for a local observer immersed in a universe filled with radiation and in the presence of a small cosmological constant. An argument for the bound might be based on the theorem [5] that states that there is an upper bound on the entropy and hence the "mass" of a black hole in de Sitter space. Such a black hole has a maximum "size" $\lambda^{-1/2}$. We have seen earlier that in the absence of a cosmological constant, the entropy that radiation in equilibrium can fit into a sphere of size $\lambda^{-1/2}$, before the onset of black hole formation is of $O(\lambda^{-3/4})$. Any more entropy stored in radiation would need more space or be repackaged. The cosmological constant merely acts as a cavity which the radiation fills.

Before concluding it would be important to describe the observation of a local observer immersed in a geometry determined by a perfect fluid and a positive cosmological constant, without using a "star" to obtain the bound. Although this gauge, known as the "O-gauge", [16] can be constructed, the technical difficulty (for us) in this gauge is that the dynamics is described by two dimensional partial differential equations.[5] This is in contrast with our use of a "star" where only one dimensional differential equations are involved.

[5]See the appendix

5. Conclusions

Let us briefly recapitulate our main message: a local observer in a four dimensional FRW universe with a positive cosmological constant will never "see" more entropy in the microwave background than $S \sim \lambda^{-3/4}$. Any excess entropy up to $O(\lambda^{-1})$ gets processed into black holes or gets absorbed by the horizon. If we ascribe the observed acceleration of our universe to a cosmological constant then the entropy in the microwave background must be smaller than $O(10^{91})$.

The local observer does not in general witness catastrophic events, he or she merely sees the microwave background and the occasional black hole. The experience of the observer becomes drastically different in the collapsing phase if the amount of entropy exceeds the "Bekenstein bound" of $O(\lambda^{-1})$, he or she ends with the rest of the universe in a big crunch.

The question of why the entropy in the microwave background is $O(10^{85})$ which is some five orders of magnitude smaller than the bound discussed in this paper remains unanswered. A plausible answer is that if there is an excess entropy produced, in reheating the universe, the process of repackaging that excess entropy somehow overshoots the bound. In this context it is interesting to note that the current estimate of the entropy stored in black holes at the center of galaxies is $O(10^{95})$. Another possible explanation in the context of inflation is that the subsequent reheating of the universe produces an amount of entropy short of the bound.

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A. The O-gauge

In order to describe what a local observer experiences, it is useful to work in the "O-gauge" coordinate system which only covers the causal region that the observer. It was shown in [16] that given a space-time metric of the FRW form

$$ds^2 = dt^2 - a(t)^2 \left(d\theta^2 + \sin^2\theta \; d\Omega_2^2\right), \tag{A.1}$$

where $-\infty < t < \infty$ and $0 \leq \theta \leq \pi$, one can make the following coordinate transformation to a coordinate system that covers only the causal patch with a metric of the form

$$ds^2 = e^v(dt^2 - dr^2) - Z^2 d\Omega_2^2, \tag{A.2}$$

where v and Z are in general functions of space and time.

First, conformally rescale (A.1) by defining the new time coordinate $\tau = \int dt/a(t)$. The Penrose diagram associated with the new metric is depicted below. For a stationary observer at $\theta = 0$, the causal patch is marked by the shaded region. A point P in the causal

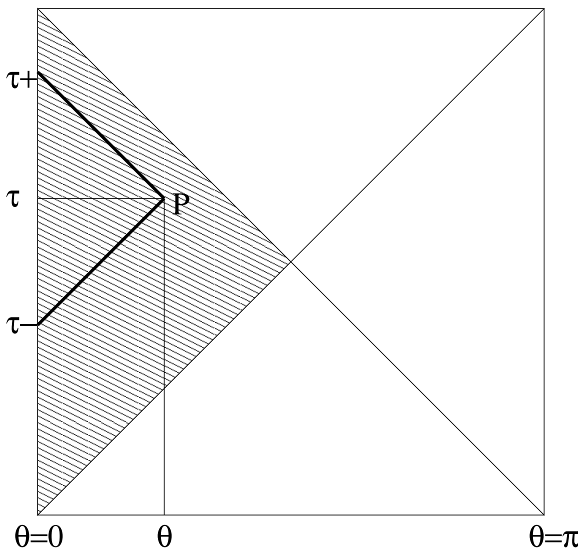

Figure 1: Penrose diagram of (A.1)

patch can be mapped to a pair of points on the observer's world-line by following the light-like trajectories to τ^+ and τ^-. Clearly, the following relations hold $\tau = (\tau^+ + \tau^-)/2$ and $\theta = (\tau^+ - \tau^-)/2$. From this it follows that the metric on the causal patch can be written as

$$ds^2 = a^2\left(\frac{\tau^+ + \tau^-}{2}\right) d\tau^+ d\tau^- - a^2\left(\frac{\tau^+ + \tau^-}{2}\right) \sin^2\left(\frac{\tau^+ - \tau^-}{2}\right) d\Omega_2^2\,, \qquad \text{(A.3)}$$

where $\tau_{\min} \leq \tau^\pm \leq \tau_{\max}$. Applying a further (holomorphic) coordinate transformation $\tau^\pm \to X^\pm(\tau^\pm) = t \pm r$, subject to the gauge-fixing conditions $v(r = 0) = Z(r = 0) = 0$ brings the metric to the form (A.2).

We will now write Einstein's equations in the presence of a perfect fluid with the equation of state $p = \kappa\rho$ and a positive cosmological constant . The energy-momentum tensor of this fluid is given by

$$T^{\mu\nu} = \rho\left((1+\kappa)U^\mu U^\nu + \kappa g^{\mu\nu}\right), \qquad \text{(A.4)}$$

where U is the 4-velocity of the fluid. The tt, rr, and tr components of Einstein's equation read

$$\begin{aligned} \frac{\dot Z\dot v}{Z} + \frac{\dot Z^2}{Z^2} + \frac{v'Z'}{Z} - 2\frac{Z''}{Z} + \frac{e^v}{Z^2} - \frac{Z'^2}{Z^2} - \lambda e^v &= T_{tt}\,, \\ \frac{\dot Z\dot v}{Z} - \frac{\dot Z^2}{Z^2} - 2\frac{\ddot Z}{Z} + \frac{v'Z'}{Z} - \frac{e^v}{Z^2} + \frac{Z'^2}{Z^2} + \lambda e^v &= T_{rr}\,, \\ \frac{\dot Z v'}{Z} + \frac{Z'\dot v}{Z} - 2\frac{\dot Z'}{Z} &= T_{tr}\,. \end{aligned} \qquad \text{(A.5)}$$

The last two components, $\theta\theta$ and $\phi\phi$ give the same equation (assuming spherical symmetry)

$$e^{-v}\left(Z\ddot Z - ZZ'' + \frac{1}{2}Z^2\ddot v - \frac{1}{2}Z^2 v'' - e^v\right) + \lambda Z^2 = T_{\theta\theta}\,. \qquad \text{(A.6)}$$

The system is further constrained by energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$. One must now give initial conditions on a space-like hyper-surface $t = t_0$ for the geometry and the fluid. The entropy on such an initial slice is given by

$$S(t = t_0) \sim \int dr\, e^{v/2} Z^2 \rho^{\frac{1}{1+\kappa}} \,. \tag{A.7}$$

What, in this gauge, would be the signal that the bound has been violated? [17].

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